Reply to Dr. Hugh Jones, Imperial College, London.

(1) *Where does the least time come into the equation*?

We start with a Lagrangian, which is given in the form:

$$S = \int_{t_0}^{t_f} (\mathfrak{L}_T + \mathfrak{L}_S + \mathfrak{L}_C)\,\mathrm{d}t$$

where

$$\mathfrak{L}_T = \frac{1}{\Delta E}\frac{\mathrm{d}s}{\mathrm{d}t} \;\; ; \quad \begin{cases} \frac{\mathrm{d}s}{\mathrm{d}t} = \sqrt{<\dot{\psi}|(1-\hat{P})|\dot{\psi}>} \\ \Delta E^2 = <\psi|(\tilde{H} - <\tilde{H}>\mathbf{1})^2|\psi> \end{cases}$$

$$\mathfrak{L}_S = +i(<\phi|\dot{\psi}> - <\dot{\psi}|\phi>) - (<\phi|\tilde{H}|\psi> + <\psi|\tilde{H}|\phi>);$$

$$\mathfrak{L}_C = \sum_j \lambda_j\, f_j(\tilde{H})$$

and $\frac{\mathrm{d}s}{\mathrm{d}t} = \Delta E$, which means the first term has the form $\int dt$, thereby justifying our claim of time optimality.

(2) *The standard brachistochrone problem concerns a constant Hermitian Hamiltonian. We widened the framework to include non-Hermitian Hamiltonians, but your Hamiltonian is Hermitian and time dependent, which I imagine gives one a great deal of latitude.*

In fact, by the geodesic theorem, the Hamiltonian is Hermitian and time dependent, which gives the operator a great deal of **longitude** in SU(2), as the geodesic on this manifold is the great circle arc! Lines of latitude are not geodesic!!

(3) *Why do you put on the constraint condition Tr(HF)=0? Is there any physical meaning to this? The standard constraint was the physical condition that the dispersion of the energy be fixed, but I don't see any physical justification for your constraint.*

The constraint comes from the requirement that the inaccessible degrees of freedom be orthogonal to the Hamiltonian operator. In the matrix vector space, the dot product is given by the Trace formula, i.e. $<\hat{A},\,\hat{B}> = \mathrm{Tr}(\hat{A}\hat{B})$. It is a working hypothesis, and it is the simplest example of a linear constraint that may be taken as a Lagrange multiplier.

(4) *Along with 1) this is my most serious objection. The "quantum brachistochrone equation" is WRONG. When one has explicit time dependence the Heisenberg equation for operators is* $\mathrm{d}/\mathrm{d}t(\hat{A}) = \partial\hat{A}/\partial t + i[\hat{H},\hat{A}]$. *You have only to consider the case A=H to see why this has to be the case.*

In fact, the quantum brachistochrone is not WRONG; it is derived from the working hypothesis. There may be some confusion as to the writing of the partial derivatives; in the text we write a full time derivative instead of partials, as the operators in question are explicit functions of time.

In your notation I would probably write:

$$i\frac{\partial}{\partial t}(\tilde{H}(t) + \tilde{F}(t)) = \tilde{H}\tilde{F} - \tilde{F}\tilde{H}$$

but since in my examples I never need to consider implicit time dependence, it is acceptable to write the full derivative sign, as long as it is understood in this form.

(5) *In the SU(2) case, which I have looked into in some detail, Ω is not defined.*

Ω is a constant that comes from the constraint $\mathrm{Tr}(\tilde{H}\tilde{F}) = 0$. In this case, we have the Hamiltonian as a time dependent linear combination of x- and y- matrices, leaving the constraint as a multiple of $\hat{\sigma}_z$. We write this as the simple formula:

$$\tilde{F} = \Omega(t)\hat{\sigma}_z$$

and after substitution into the quantum brachistochrone equation, we find that $\dot{\Omega} = 0$. In fact, the final calculated result gives us the simple expression:

$$\Omega T = \frac{n\pi}{2} \quad n \in \mathbb{N}.$$

where

$$|\Psi(0)> = \frac{1}{\sqrt{2}}[1,1]^T, \quad |\Psi(T)> = \frac{1}{\sqrt{2}}[1,-1]^T$$

(6) *Again, the quantum brachistochrone equation as written is WRONG!*

Sorry, but I object to your vigorous statement on the simple observation that it has no basis whatsoever. This is a physically motivated schemata; in fact the derived results on SU(2) and SU(3) would seem to indicate that the method *derives* the physical operators....

But that makes sense, as nature hates a time waster, and it is natural to consider a flow that is time optimal, after all it is in the path that lightning takes from a cloud to the ground, the path that water flows down from the top of a mountain to the sea, and many other things.

Hope that is of help in addressing your concerns.

Yours sincerely,

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