Clifford Algebras, Permutations and Quantum Physics

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Abstract:

The possible transformations that can occur between inputs and outputs in a quantum computer interface are examined. Problems discussed include the proposition of initial condition for such a device, the effects of noise on the efficient operation of the system, and the concept of entanglement as it applies to quantum computation and algorithmics. A useful method of representing quantum binary operations is presented and a method of writing the Hamiltonian connecting past and future is outlined which has some very interesting properties. Some theorems relating to spanning sets over matrices with complex-valued entries are proven.

Introduction:

The future of computation rests with the success or failure of quantum computation. With the extrapolated shrinkage of size in chipsets over the next 10 years it is important to consider quantum effects not only in the operation of quantum computer systems, but also in potential problems they might cause within classical computer systems that exist today. At present it appears difficult to manufacture scaleable, reliable and replicable two qubit quantum computers; several reliable one-qubit systems are already in operation.

The most apparent difficulty with the operation of any computer system is the dissipation of heat. This is intimately related to the physical process of information creation and destruction, which in turn is related to the 2nd Law of Thermodynamics. The proper place for entropy and information dynamics within quantum physics remains unclear; in fact quantum entropy has a variety of measures and differing properties to the classical limit. Just how one reduces to the other and the relation between the two is an open problem. The processes of information exchange, entropy creation and heat dissipation all require some interaction between the system and environment, so it is important to analyse the physical nature of such processes.

Quantum physics still retains vestiges of an old philosophy, sometimes referred to as the "Copenhagen intepretation" that maintains a division between the experimenter and the experiment proper. The experiment, considered to be an atom in a box, has properties that we term "quantum", and the experimenter, represented by an external potential, is considered to be a classical or continuous object with no quantum properties. This implies that there is an asymmetry between the observer and observed, that measurement by the experimenter causes no effect on their behaviour.

Results of the analysis detailed indicate some areas in which this distinction may be weakened and removed in order to maintain a greater symmetry in the transformations between the system and environment. By considering the operation of a two-qubit device, and allowing the interaction of the observer in initialisation, measurement and data flow to be analysed, the Clifford Group emerges as a natural way of describing the dynamics of the system in a binary matrix representation. Indeed, on quite simple hypotheses it is possible to conclude that when the experiment is over, it isn't over: it has only just begun again!

Section 1: Matrix Representation of Qinary Operators

Consider two input known photons scattered by an initial known atom in a fixed state. Only polarisation effects are considered; future papers will cover the energy-momentum results. In order to carry out the gedankenexperiment consistently according to quantum mechanics it is necessary to consider all possible experimental results in order to obtain a complete set of base states.

There are four possible basis states depending on the spin states of the photons and atom; these are the quantum-binary logical states which will be labelled as $|00\rangle, |01\rangle, |10\rangle$ and $|11\rangle$. The atom itself has these states as well; the transformations for the photonic and atomic systems that are symmetric for both are those that are to be considered.

Writing the state vector as

$$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle = \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix}$$

it is apparent that the transformations to be used are the 4 x 4 matrices; what is required is a spanning set of quantum operations that can be used to implement the computation. At present the focus in the literature is on single-qubit operations; this paper takes a different perspective in examining the matrices that operate upon all four qubits together as a whole.

The first transformation to consider is [CNOT1], defined by the logical table:

$$\begin{array}{l} |00\rangle \rightarrow |00\rangle \\ |01\rangle \rightarrow |01\rangle \\ |10\rangle \rightarrow |11\rangle \\ |11\rangle \rightarrow |10\rangle \end{array} \quad \text{which can be represented as the matrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \text{on our state vector.}$$

This can also be written out as a logical operation within an arbitary basis vector, given by:

$$[\text{CNOT1}]|a; b\rangle = |a;\ \text{if } a = 0 \text{ then do } b \rightarrow b, \text{else do } b \rightarrow \text{not}(b)\rangle$$

This is of the form "If Condition 1 , Do Property 1, Else Do Property 2". Consequently the question arises whether there are any other matrices with symmetries to [CNOT1] that can be written in this way. There are, and the relevant transformations are given by:

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

henceforth to be known as [CNOT2], [CNOT3] and [CNOT4] respectively. These matrix representations can be written out as basic logical algorithms as outlined below.

Procedure for CNOT3

Input = (IN1 ; IN2): Output = (O1 ; O2)

IF IN1 = 0 THEN DO Output = (NOT(IN1) ; NOT(IN2))

ELSE DO Output = (NOT(IN1) ; IN2)

Also it is important to note that these matrices act as operations on spinor state vectors. This is due to the particular symmetry of this choice of matrices. As these all have algorithmic decompositions of the form:

IF (Property x) THEN DO (Operation A) ELSE DO (Operation B)

it is relevant to consider what other types of basic logical transformations are possible on the qinary bits $|00\rangle, |01\rangle, |10\rangle$ and $|11\rangle$. One type of fundamental computer program or logical operation that is quite central to the operation of most software systems is the command:

IF (Property x) THEN DO .. LOOP

This is similar to the CNOT structure; however, instead of two chains of two vectors there is a a permutation cycle.

Noting also that some of the CNOT matrices have this form (CNOT3 and CNOT4) it is found that the relevant matrices to complete the group of transformations, both in the 2-member cycles and 4-member cycles are given by the set:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

which are henceforth referred to in this text as [LOOP1], [LOOP2], [LOOP3] and [LOOP4].

LOOP2, LOOP4, CNOT3 and CNOT4 have a 4-cycle structure; LOOP1 and LOOP3 together with CNOT1 and CNOT2 have a 2-cycle structure. The symmetry of these matrices is quite striking, and is readily seen to correspond to a movement inwards or outwards from the centre by one unit to move between CNOT and LOOP; within each set the members are related by physical rotations of the matrices by $\frac{\pi}{2}$.

These matrices have a number of interesting properties, including such formulae as:

$$\big[\mathrm{CNOT1}\big]^2 = \big[\mathrm{CNOT2}\big]^2 = \big[\mathrm{LOOP1}\big]^2 = \big[\mathrm{LOOP3}\big]^2 = I \tag{1}$$

$$\big[\mathrm{CNOT3}\big]^2 = \big[\mathrm{CNOT4}\big]^2 = \big[\mathrm{FLIP1}\big] \tag{2}$$

$$\big[\mathrm{LOOP2}\big]^2 = \big[\mathrm{LOOP4}\big]^2 = \Big[\mathrm{FLIP2}\Big] \tag{3}$$

where I, [FLIP1] and [FLIP2] are defined respectively by the matrices:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

The composition of the two groups results in a final set of matrices with a rather remarkable symmetry property; instead of being a straight rotation like the matrices above to move within the group, each member is related to the others by two transformations, a flip and a rotation. These can be most easily defined and generated by the multiplications:

$$\big[\mathrm{CSUB}(N)\big] = \big[\mathrm{CNOT1}\big]\big[\mathrm{LOOP}(N)\big] \quad (4) \quad \text{and} \quad \big[\mathrm{GSUB}(N)\big] = \big[\mathrm{LOOP1}\big]\big[\mathrm{CNOT}(N)\big] \tag{5}$$

where N is the label of the index, being either 1, 2, 3 or 4. These matrices are given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

These matrices have a logical operation upon the input qinary bit that contains a 3-member cycle and a 1-member cycle. Of course, a 1-member cycle maps bits to themselves.

Multiplication of the CSUB and GSUB matrices shows that they are closed under plain matrix multiplication within themselves; together with the functions FLIP1, FLIP2, I and NOT they form a closed group, as these are composed by multiplying a GSUB with a CSUB matrix. Multiplication of LOOP(N) with CSUB4 generates 4 more matrices to complete the permutation group. They are referred to as SWAP1, SWAP2, FLOP1 and FLOP2 and are given by:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Section 2: Applications of Qinary Operators in Simulations

Consider our state vector to be an explicit function of time, and we allow the system to evolve freely between the time when it is initialised and when it is measured under an operator $H(t)$ then the system will follow the Heisenberg equation:

$$\left|\psi(t)\right\rangle = \exp(-i\int_0^t \mathrm{ds}\cdot H(s))\left|\psi(0)\right\rangle \tag{6}$$

We now wish to calculate how a state with some initial condition will evolve under CNOT, LOOP, GSUB and CSUB type operations, to simulate how a quantum circuit with these components will behave. Using power series expansions and equating real and imaginary parts it is quite straightforward to obtain the results:

$$\exp(-i\int_0^t \mathrm{ds}\cdot[Y](s)) = \cos(t)[I] - i\cdot\sin(t)[Y] \tag{7}$$

where [Y] is one of [CNOT1], [CNOT2], [LOOP1], [LOOP3], [NOT], [FLIP1], [FLIP2], [SWAP1], [SWAP2] or [I].

These matrices are the only ones in the group that evolve in this fashion; the remainder evolve in a fashion that is order dependent in the power series. To see this, consider the exponent of [LOOP2]:

$$\begin{aligned} \exp(-i[\mathrm{LOOP2}]t) &= [I] + (-i)[\mathrm{LOOP2}]t + \tfrac{1}{2!}(-i)^2[\mathrm{LOOP2}]^2t^2 + \ldots\ldots \qquad (8)\\ &= (1+\tfrac{1}{4!}(-i)^4t^4+\ldots)[I] \\ &\quad + ((-i)t + \tfrac{1}{5!}(-i)^5t^5+\ldots)[\mathrm{LOOP2}] \\ &\quad + (\tfrac{1}{2!}(-i)^2t^2 + \tfrac{1}{6!}(-i)^6t^6+\ldots)[\mathrm{FLIP1}] \\ &\quad + (\tfrac{1}{3!}(-i)^3t^3 + \tfrac{1}{7!}(-i)^7t^7+\ldots)[\mathrm{LOOP4}] \\ &= F(t)[I] + G(t)[\mathrm{FLIP1}] + i(J(t)[\mathrm{LOOP2}] + K(t)[\mathrm{LOOP4}]) \qquad (9) \end{aligned}$$

where $F(t), G(t), J(t)$ and $K(t)$ are real functions of time defined by the power series given above.

To calculate transition elements and expectation values using this formalism is straightforward. As a demonstration, examine the expectation value of a [NOT] operator.

Firstly, using (6) and (7) we find that:

$$\left|\phi_{\mathrm{NOT}}\right\rangle_{(t)} = \cos(t)[I]\left|\phi_{\mathrm{NOT}}\right\rangle_{(0)} - i\cdot\sin(t)[\mathrm{NOT}]\left|\phi_{\mathrm{NOT}}\right\rangle_{(0)} \tag{10}$$

and following it through we obtain the result:

$$\langle\varphi(t)|[\mathrm{NOT}]|\phi(0)\rangle = \cos(t)\langle\varphi(0)|[\mathrm{NOT}]|\phi(0)\rangle + i\cdot\sin(t)\langle\varphi(0)|[I]|\phi(0)\rangle \qquad (11)$$

Calculating the transition elements using orthogonality and logic operations, the only non-zero components are:

$$\langle 00(t)|[\mathrm{NOT}]|00(0)\rangle = i\cdot\sin(t)$$

$$\langle 11(t)|[\mathrm{NOT}]|00(0)\rangle = \cos(t)$$

for the specific input 00. There are 2 elements that correspond to these for each other input, given by the qinary values of 01, 10 and 11. It is interesting to note that the first amplitude gives the probability that no calculation occurs, whereas the second gives the amplitude that the correct calculation was conducted. To ensure that we return the "correct" answer, it is important that the output is measured around the times $\cos(t) = 1$, that is $t = n\pi$.

This shows that the output from the quantum computer, at least for this operation, is a function of the time. A classical computer would be expected to output the same answer given a constant input set; however with the quantum computer we see that the probability of obtaining the 'correct' result oscillates as a function of time, and consequently the measurement time is important in the output of the calculation.

Section 3: Transition Elements and Time-Ordered Operators

For composite operations, such as the series operation of two different logic functions, say A and B, a theorem due to Feynman is useful. This method of treatment of ordered operators, called "disentangling an operator" involves adding an extra indice to the operator that indicates the time of application. Feynman [1] states that the "the solution of such problems might open up the possibility of a true infinitesmal calculus of quantities in the field of hypercomplex numbers" when referring to the disentanglement problem of the Dirac matrices.

It is relevant to note that the calculation of Heisenberg S-matrix elements is much easier in this matrix representation against the Dirac representation. The important point is that, although calculation in the S-matrix picture is simple and straightforward, the equation that should be examined extremely closely is the path integral form.

Now, consider a time-ordered sequence of these operators. The symbol $\varphi_{\mu\nu}$ represents any of the operators from CNOT(N), LOOP(N), GSUB(N), CSUB(N), FLIP(N), FLOP(N), NOT or I. This is extended such that we use the notation $\varphi_{\mu\nu}(t_2, t_1)$ to mean the operator $\varphi_{\mu\nu}$ acting on the input between times t_1 and t_2.

We can write that the state vector evolves in time according to the law:

$$\left|\Psi(t_2)\right\rangle = \exp(-i\int_{t_1}^{t_2}\mathrm{ds}\cdot\varphi_{\mu\nu}^{-+})\left|\Psi(t_1)\right\rangle = \hat{S}^{-+}(t_2,t_1)\left|\Psi(t_1)\right\rangle$$

a related expression:

$$\left|\Psi(t_1)\right\rangle = \exp(i\int_{t_2}^{t_1}\mathrm{ds}\cdot\varphi_{\mu\nu}^{--})\left|\Psi(t_2)\right\rangle = \hat{S}^{--}(t_1,t_2)\left|\Psi(t_2)\right\rangle$$

and the adjoint equations given by:

$$\langle\Psi(t_2)| = \langle\Psi(t_1)|\,\exp(i\int_{t_1}^{t_2}\mathrm{ds}\cdot\varphi_{\mu\nu}^{+-}) = \left\langle\Psi(t_1)\right|\hat{S}^{+-}(t_2,t_1)$$

$$\left\langle\Psi(t_1)\right| = \left\langle\Psi(t_2)\right|\exp(-i\int_{t_2}^{t_1}\mathrm{ds}\cdot\varphi_{\mu\nu}^{++}) = \left\langle\Psi(t_2)\right|\hat{S}^{++}(t_1,t_2)$$

Then the most apparent path integral to write down is the expression:

$$G(\phi_{\rm out}, t_{\rm out} \big| \phi_{\rm in}, t_{\rm in}) = \int \ldots \int d\big[\Phi(t)\big] \exp(-i \int_{t_{\rm in}}^{t_{\rm out}} {\rm ds} \cdot T(\varphi_{\mu\nu}(s)))$$

where the symbol in the exponential indicates a time-ordered operator over the field of the quantum logical matrices, and the configuration sum is taken over all intermediate states.

To use this expression, note that the order of operation of the logical functions upon the input state is important as the matrices do not necessarily commute; write down the possible intermediate states and relevant matrix operations, then sum over all possible permutations between the initial and final states to obtain the transition amplitude.

For example, if the program "DO [LOOP2]AND/OR[CNOT1]" was desired to be run on our quantum computer, the order of operation of CNOT1 and LOOP2 is important as they do not commute. Then we can write the transition amplitude as:

$$G(\phi_2, t_2; \phi_1, t_1) = \int \ldots . \int d[\phi(t)] \exp(-i \int {\rm ds} \cdot T({\rm CNOT1}(s), {\rm LOOP2}(s)))$$

where the index indicates the time ordering of the series. These terms in the transition element can be written out explicitly with appropriate limits over the integral; these limits are taken over all intermediate times. Note that from our definitions and appropriate choices of the operators from the 24 matrices available automatically yields the properties that:

$$\hat{S}^{+-}\hat{S}^{--} = G(\phi_2, t_2; \phi_1, t_1) \qquad \text{as well as} \qquad \hat{S}^{++}\hat{S}^{-+} = G(\phi_1, t_1; \phi_2, t_2) \quad t_2 > t_1$$

The convention is also made that the calculation is always in motion, such that the amplitude for the input to be static in time is zero. This results in the orthogonality relationships:

$$\hat{S}^{++}\hat{S}^{--} = \hat{S}^{+-}\hat{S}^{-+} = 0$$

These formulae simply state that the amplitude of a dually-entangled system-environment or a null experiment can be taken as zero, for they produce no observable effects at that exact time until they are communicated to an impartial observer.

Section 4: Physical Interpretation of the Matrix Operators

As there are N!=4!=24 matrices, all real valued, it makes sense to enquire which of these form a spanning set for the 4x4 matrices. As there are obviously 8 extra linearly dependent members this is not a basis. However, it is interesting to note that there is an expansion of the form:

$$\hat{S}(t, [\varphi_{\rm jk}]) = \sum_n f_{\rm jn}(t) \varphi_{\rm nk}$$

where the co-efficient is a complex valued function of time. If we examine the time-reversed transform it is apparent that the symmetry of the situation requires 8 matrices to be future vectors, 8 to be the past vectors and 8 vectors representing the transformation between them.

Writing this out in terms of the matrix operators:

$$\hat{\varphi}_{\mu\nu} = \tfrac{1}{2}(\hat{\varphi}^{\,f}_{\mu\nu} + \hat{\varphi}^{\,p}_{\mu\nu}) + \hat{\varphi}^{\rm int}_{\mu\nu}$$

and substitution into the Schrodinger equation yields the form:

$$\hat{H}_{\rm tot} = \hat{H}_f + \hat{H}_p + \hat{H}_{\rm int}$$

where the indices represent future, past and the interaction between them.

A physical interpretation of this is thus: the requirement of 8 past states and 8 future states is to be intimately linked with the nature of an entangled photon pair. Each of the photons composing the pair can be in 4 possible states at any one time, being IN- OUT- LHC- and RHC-polarised. This physical meaning is straightforward to see from the above equations, as the matrix operator has a separation exactly symmetrical to the separation of the potential operators in quantum electrodynamics into advanced and retarded fields. The form of the Hamiltonian operator also enables all the techniques of Feynman diagrams to be directly translated and used for this application, as it is directly equivalent in structure to two interacting systems joined by a mutual field. The field in this context is the "field of polarisation" which for the entangled photon pair has deeply non-local characteristics. It is interesting to note that the entangled pair is never actually observed, even though its prescence has tangible effects. Also, since the structure of the Hamiltonian is so similar to that employed in quantum electrodynamics it will be interesting to examine whether the matrix operators can be employed in this area to simplify calculations.

Finally, the transformed state after some small increment of time is given by:

$$\left|\Psi(t+\epsilon)\right\rangle = \exp(-i\epsilon\hat{H}(t,0))\left|\Psi(t)\right\rangle$$

and iterating this process and using the expression again, we obtain:

$$\left|\Psi(t+2\epsilon)\right\rangle = \exp(-i\epsilon\hat{H}(t,1))\left|\Psi(t+\epsilon)\right\rangle + \hat{S}(2,0)\left|\psi(t)\right\rangle = \left[\hat{S}(2,0)+\hat{S}(2,1)\hat{S}(1,0)\right]\left|\Psi(t)\right\rangle$$

from which it can be concluded that when the simulation is complete at one time, it iterates until the next increment of time. As we have made the convention that the calculation is always in motion, and the calculation is simulating some physical experiment, we can make a striking observation: that when the experiment is over, it has only just begun again! That this is true can be seen in the context of data from past experiments feeding back and creating future experiments for the scientist working in the laboratory.

Conclusion:

The results detailed demonstrate that it is possible to represent physical Hamiltonians of time ordered quantities using permutation matrices. This matrix group emerges as a natural representation, and is physically equivalent to calculating the coupling between system and environment to be an exchange process of entangled photon pairs. This type of interaction has a high degree of symmetry, and also a remarkable property that the entangled photon pair is never actually observed in its totality; we either observe one photon or the other. This non-locality property needs to be examined in greater depth.

Implications for Quantum Computing Systems:

Consider a quantum system, given by two atoms inside a dot, and an external environment represented by two atoms on the end of an atomic force microscope probe. The difficulty with running a quantum computer appears to be one of initialisation and resonance. What guarantee is there that the two atoms are in the same initial state that the calculation was started with? For this would require the system to be sealed from the environment with certainty in order to prevent any interaction whatsoever, which is not quantum mechanically possible.

Instead, consider an alternative scenario where the box is initialised using a magnetic field, data is input as an entangled photon pair, temperature of the bath is measured in order to find the degree of error, and output is another entangled photon pair. The interested reader will be able to show that the transformations that are symmetric between input/output and system/observer are the matrix group used in this paper. As such, it is important that the entire system and environment is resonant; by this it is implied that the data flow through the device can be maximised by not constantly measuring the temperature variable continuously. Rather it would seem to make a lot more physical sense to switch between INITIALISE/INPUT and CHECK/OUTPUT at the resonant frequency of the device.

Appendix 1:

Hermitian Real-Valued Matrices

Let $\hat{M} = \hat{A} + i\hat{B}$ be Hermitian, with $\hat{A}, \hat{B}$ real matrices.

Then by definition $\hat{M}^* \hat{M} = \hat{M}\hat{M}^* = \hat{I}$ where I is the identity matrix and the star indicates the Hermitian conjugate, being complex conjugate and transposition.

For this to be a closed group, we also require all products under matrix multiplication to be members of the group. That is:

$$\text{Given } \hat{M}_i, \hat{M}_j \in G, \left[\hat{M}_i, \hat{M}_j\right] \text{ and } \left\{\hat{M}_i, \hat{M}_j\right\} \text{exist and are of the same form as } \hat{M}_i, \hat{M}_j$$

where the brackets indicate the commutator and anticommutator as normal. Examination of the requirement for hermiticity results in the expressions:

$$\hat{A}^T\hat{A} + \hat{B}^T\hat{B} = \hat{A}\hat{A}^T + \hat{B}\hat{B}^T = I \qquad \text{and} \qquad \hat{A}^T\hat{B} - \hat{B}^T\hat{A} = \hat{A}\hat{B}^T - \hat{B}\hat{A}^T = 0$$

which can be neatly summarised in the equations:

$$_{(T)}\left\{\hat{A}, \hat{B}\right\} = \left\{\hat{A}, \hat{B}\right\}_{(T)} = I \qquad \text{and} \qquad _{(T)}\left[\hat{A}, \hat{B}\right] = \left[\hat{A}, \hat{B}\right]_{(T)} = 0$$

where the index indicates on which order (first or second) the transposition operation occurs. The set of matrices given by the quantum-logical operators are not a basis; it is possible to say that they form a closed group under ordinary matrix multiplication as every element multiplied against every other element gives no new vector. This is a simple exercise to conduct, even on a classical computer!

Now consider a new element, given by the square of the element. We wish to show that if we are given a basis set of elements for the group, that this new element given by the square of an element is still within the span of the group.

Assume that within our set of 24 matrices that we have a spanning set; the dimension of this spanning set is 16 by simple consideration.

Now $\hat{M}^2 = \hat{M}' \in \text{span}(G)$ is what is required. A brief calculation shows that:

$$\hat{M}' = \hat{A}^2 - \hat{B}^2 + i\left\{\hat{B}, \hat{A}\right\} = \hat{C} + i\hat{D} \in \text{span}(G) \quad \text{for all} \quad \hat{A}, \hat{B} \in M^{\mathbb{R}}_{4\text{x}4} = \text{span}(G)$$

The basis set for the quantum logical operators has not been determined thus far. Further papers will outline the results and conclusions that stem from this calculation. A brief examination of the properties of the basis set should demonstrate their separation into scalar, vector, tensor, axial vector and pseudoscalar components as required.

Appendix 2: Proof Of Equivalence With Lindblad Equation

Consider the path integral form of the S-matrix operator. The most general formula that can be given involving coupling of two electromagnetic fields with an atom is:

$$\hat{G}(t,0) = \left\langle \gamma_\lambda(t), t \middle| \gamma_\nu(0), 0 \right\rangle = \int_{\text{all paths}} \int d[F^{\mu\nu}(t)] \cdot \exp\Big(-i \int_0^t \mathrm{ds} \cdot \Big[\gamma_\nu F^{\nu\lambda}, \gamma_\lambda\Big] + \Big[\gamma_\nu, F^{\nu\lambda}\gamma_\lambda\Big](s)\Big)$$

Making the identifications of the necessary parts:

$$\hat{\rho}(t) = F^{\nu\lambda}(t) = \hat{G}(t,0) \qquad \text{and} \qquad \gamma_\lambda \to \hat{\sigma}_+, \gamma_\nu \to \hat{\sigma}_- \text{ with } \hat{G}(0,0) = \hat{I}$$

to obtain the non-relativistic limits of the expression, one immediately can write down that:

$$i\delta\hat{\rho}/\delta t = \Big[2\hat{\sigma}_- \cdot \hat{\rho} \cdot \hat{\sigma}_+ - \Big\{\hat{\rho}, \hat{\sigma}_+\hat{\sigma}_-\Big\}\Big]$$

which is the Lindblad Master Equation in the interaction picture. Note however that the path integral form (which is equivalent in all respects to the S-matrix operator) is of a general nature whereas the master equation is of restricted application. Also note that the energy-momentum tensor is specifically related to the density matrix; this is to be expected as the energy and momentum values that the field can take are derived from the eigenvalues of this quantity. The hypercomplex amplitude in the exponent is given by the set of quantum-logical matrices used throughout this paper, with appropriate time orderings as required.

References:

[1] R.P. Feynman, *Operator Calculus Having Applications in Quantum Electrodynamics.*

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